

Scotogenic Inverse Seesaw Model of Neutrino Mass

Sean Fraser, Ernest Ma, and Oleg Popov

*Department of Physics and Astronomy, University of California,
Riverside, California 92521, USA*

Abstract

A variation of the original 2006 radiative seesaw model of neutrino mass through dark matter is shown to realize the notion of inverse seesaw naturally. The dark-matter candidate here is the lightest of three real singlet scalars which may also carry flavor.

In 1998, the simplest realizations of the dimension-five operator [1] for Majorana neutrino mass, i.e. $(\nu_i \phi^0)(\nu_j \phi^0)$, were discussed systematically [2] for the first time. Not only was the nomenclature for the three and only three tree-level seesaw mechanisms established: (I) heavy singlet neutral Majorana fermion N [3], (II) heavy triplet Higgs scalar (ξ^{++}, ξ^+, ξ^0) [4], and (III) heavy triplet Majorana fermion $(\Sigma^+, \Sigma^0, \Sigma^-)$ [5], the three generic one-loop irreducible radiative mechanisms involving fermions and scalars were also written down for the first time. Whereas one such radiative mechanism was already well-known since 1980, i.e. the Zee model [6], a second was not popularized until eight years later in 2006, when it was used [7] to link neutrino mass with dark matter, called *scotogenic* from the Greek *scotos* meaning darkness. The third remaining unused mechanism is the subject of this paper. It will be shown how it is a natural framework for a scotogenic inverse seesaw model of neutrino mass, as shown in Fig. 1. The new particles are three real singlet scalars $s_{1,2,3}$, and one set

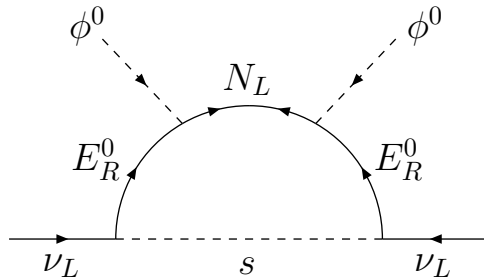


Figure 1: One-loop generation of inverse seesaw neutrino mass.

of doublet fermions $(E^0, E^-)_{L,R}$, and one Majorana singlet fermion N_L , all of which are odd under an exactly conserved discrete symmetry Z_2 . This specific realization was designated T1-3-A with $\alpha = 0$ in the compilation of Ref. [8]. Note however that whereas $(E^0, E^-)_L$ is not needed to complete the loop, it serves the dual purpose of (1) rendering the theory to be anomaly-free and (2) allowing E to have an invariant mass for the implementation of the inverse seesaw mechanism.

The notion of inverse seesaw [9, 10, 11] is based on an extension of the 2×2 mass matrix

of the canonical seesaw to a 3×3 mass matrix by the addition of a second singlet fermion. In the space spanned by (ν, N, S) , where ν is part of the usual lepton doublet (ν, l) and N, S are singlets, all of which are considered left-handed, the most general 3×3 mass matrix is given by

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_2 & 0 \\ m_2 & m_N & m_1 \\ 0 & m_1 & m_S \end{pmatrix}. \quad (1)$$

The zero $\nu - S$ entry is justified because there is only one ν to which N and S may couple through the one Higgs field ϕ^0 . The linear combination which couples may then be redefined as N , and the orthogonal combination which does not couple is S . If $m_{S,N}$ is assumed much less than m_1 , then the induced neutrino mass is

$$m_\nu \simeq \frac{m_2^2 m_S}{m_1^2}. \quad (2)$$

This formula shows that a nonzero m_ν depends on a nonzero m_S , and a small m_ν is obtained by a combination of small m_S and m_2/m_1 . This is supported by the consideration of an approximate symmetry, i.e. lepton number L , under which $\nu, S \sim +1$ and $N \sim -1$. Thus $m_{1,2}$ conserve L , but m_S breaks it softly by 2 units. Note that there is also a finite one-loop contribution from m_N [12, 13].

Other assumptions about m_1, m_S, m_N are also possible [14]. If $m_2, m_N \ll m_1^2/m_S$ and $m_1 \ll m_S$, then a double seesaw occurs with the same formula as that of the inverse seesaw, but of course with a different mass hierarchy. If $m_1, m_2 \ll m_N$ and $m_1^2/m_N \ll m_S \ll m_1$, then a lopsided seesaw [14] occurs with $m_\nu \simeq -m_2^2/m_N$ as in the canonical seesaw, but $\nu - S$ mixing may be significant, i.e. $m_1 m_2 / m_S m_N$, whereas $\nu - N$ mixing is the same as in the canonical seesaw, i.e. $\sqrt{m_\nu / m_N}$. In the inverse seesaw, $\nu - N$ mixing is even smaller, i.e. m_ν / m_2 , but $\nu - S$ mixing is much larger, i.e. m_2 / m_1 , which is only bounded at present by about 0.03 [15]. In the double seesaw, the effective mass of N is m_1^2 / m_S , so $\nu - N$ mixing is also $\sqrt{m_\nu / m_N}$. Here $m_S \gg m_N$, so the $\nu - S$ mixing is further suppressed by m_1 / m_S .

In the original scotogenic model [7], neutrino mass is radiatively induced by heavy neutral Majorana singlet fermions $N_{1,2,3}$ as shown in Fig. 2. However, they may be replaced by Dirac

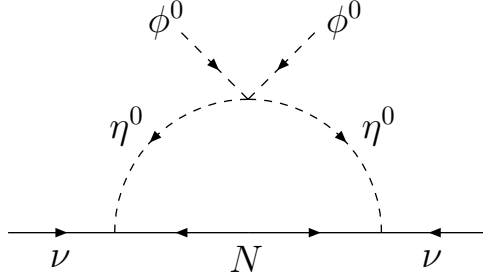


Figure 2: One-loop generation of seesaw neutrino mass with heavy Majorana N .

fermions. In that case, a $U(1)_D$ symmetry may be defined [16], under which $\eta_{1,2}$ transform oppositely. If Z_2 symmetry is retained, then a radiative inverse seesaw neutrino mass is also possible [17, 18]. We discuss here instead the new mechanism of Fig. 1, based on the third one-loop realization of neutrino mass first presented in Ref. [2]. The smallness of m_N , i.e. the Majorana mass of N_L , may be naturally connected to the violation of lepton number by two units, as in the original inverse seesaw proposal using Eq. (1). It may also be a two-loop effect as first proposed in Ref. [19], with a number of subsequent papers by other authors, including Refs. [20, 21, 22].

In our model, lepton number is carried by $(E^0, E^-)_{L,R}$ as well as N_L . This means that the Yukawa term $\bar{N}_L(E_R^0\phi^0 - E_R^- \phi^+)$ is allowed, but not $N_L(E_L^0\phi^0 - E_L^- \phi^+)$. In the 3×3 mass matrix spanning $(\bar{E}_R^0, E_L^0, N_L)$, i.e.

$$\mathcal{M}_{E,N} = \begin{pmatrix} 0 & m_E & m_D \\ m_E & 0 & 0 \\ m_D & 0 & m_N \end{pmatrix}, \quad (3)$$

m_E comes from the invariant mass term $(\bar{E}_R^0 E_L^0 + E_R^+ E_L^-)$, m_D comes from the Yukawa term given above connecting N_L with E_R^0 through $\langle \phi^0 \rangle = v$, and m_N is the soft lepton-number breaking Majorana mass of N_L . Assuming that $m_N \ll m_D, m_E$, the mass eigenvalues of

$\mathcal{M}_{E,N}$ are

$$m_1 = \frac{m_E^2 m_N}{m_E^2 + m_D^2}, \quad (4)$$

$$m_2 = \sqrt{m_E^2 + m_D^2} + \frac{m_D^2 m_N}{2(m_E^2 + m_D^2)}, \quad (5)$$

$$m_3 = -\sqrt{m_E^2 + m_D^2} + \frac{m_D^2 m_N}{2(m_E^2 + m_D^2)}. \quad (6)$$

In the limit $m_N \rightarrow 0$, E_R^0 pairs up with $E_L^0 \cos \theta + N_L \sin \theta$ to form a Dirac fermion of mass $\sqrt{m_E^2 + m_D^2}$, where $\sin \theta = m_D / \sqrt{m_E^2 + m_D^2}$. This means that the one-loop integral of Fig. 1 is well approximated by

$$m_\nu = \frac{f^2 m_D^2 m_N}{16\pi^2 (m_E^2 + m_D^2 - m_s^2)} \left[1 - \frac{m_s^2 \ln((m_E^2 + m_D^2)/m_s^2)}{(m_E^2 + m_D^2 - m_s^2)} \right]. \quad (7)$$

This expression is indeed of the form expected of the inverse seesaw.

The radiative mechanism of Fig. 1 is also suitable for supporting a discrete flavor symmetry, such as Z_3 . Consider the choice

$$(\nu_i, l_i)_L \sim \underline{1}, \underline{1}', \underline{1}'', \quad s_1 \sim \underline{1}, \quad (s_2 + is_3)/\sqrt{2} \sim \underline{1}', \quad (s_2 - is_3)/\sqrt{2} \sim \underline{1}'', \quad (8)$$

with mass terms $m_s^2 s_1^2 + m_s'^2 (s_2^2 + s_3^2)$, then the induced 3×3 neutrino mass matrix is of the form

$$\begin{aligned} \mathcal{M}_\nu &= \begin{pmatrix} f_e & 0 & 0 \\ 0 & f_\mu & 0 \\ 0 & 0 & f_\tau \end{pmatrix} \begin{pmatrix} I(m_s^2) & 0 & 0 \\ 0 & 0 & I(m_s'^2) \\ 0 & I(m_s'^2) & 0 \end{pmatrix} \begin{pmatrix} f_e & 0 & 0 \\ 0 & f_\mu & 0 \\ 0 & 0 & f_\tau \end{pmatrix} \\ &= \begin{pmatrix} f_e^2 I(m_s^2) & 0 & 0 \\ 0 & 0 & f_\mu f_\tau I(m_s'^2) \\ 0 & f_\mu f_\tau I(m_s'^2) & 0 \end{pmatrix}, \end{aligned} \quad (9)$$

where I is given by Eq. (7) with f^2 removed. Let $l_{iR} \sim \underline{1}, \underline{1}', \underline{1}''$, then the charged-lepton mass matrix is diagonal using just the one Higgs doublet of the standard model, in keeping with the recent discovery [23, 24] of the 125 GeV particle. To obtain a realistic neutrino

mass matrix, we break Z_3 softly, i.e. with an arbitrary 3×3 mass-squared matrix spanning $s_{1,2,3}$, which leads to

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & i/\sqrt{2} \\ 0 & 1/\sqrt{2} & -i/\sqrt{2} \end{pmatrix} O^T \begin{pmatrix} I(m_{s1}^2) & 0 & 0 \\ 0 & I(m_{s2}^2) & 0 \\ 0 & 0 & I(m_{s3}^2) \end{pmatrix} O \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & i/\sqrt{2} & -i/\sqrt{2} \end{pmatrix}, \quad (10)$$

where O is an orthogonal matrix but not the identity, and there can be three different mass eigenvalues $m_{s1,s2,s3}$ for the $s_{1,2,3}$ sector. The assumption of Eq. (8) results in Eq. (10) and allows the following interesting pattern for the neutrino mass matrix \mathcal{M}_ν . The Yukawa couplings $f_{e,\mu,\tau}$ may be rendered real by absorbing their phases into the arbitrary relative phases between E_R^0 and $\nu_{e,\mu,\tau}$. If we further assume $f_2 = f_3$, then \mathcal{M}_ν is of the form [25]

$$\mathcal{M}_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}, \quad (11)$$

where A and B are real. Note that this pattern is protected by a symmetry first pointed out in Ref. [26], i.e. $e \rightarrow e$ and $\mu - \tau$ exchange with CP conjugation, and appeared previously in Refs. [27, 28]. As such, it is also guaranteed to yield maximal $\nu_\mu - \nu_\tau$ mixing ($\theta_{23} = \pi/4$) and maximal CP violation, i.e. $\exp(-i\delta) = \pm i$, whereas θ_{13} may be nonzero and arbitrary. Our scheme is thus a natural framework for this possibility. Further, from Eq. (7), it is clear that it is also a natural framework for quasi-degenerate neutrino masses as well. Let

$$F(x) = \frac{1}{1-x} \left[1 + \frac{x \ln x}{1-x} \right], \quad (12)$$

where $x = m_s^2/(m_E^2 + m_D^2)$, then Eq. (7) becomes

$$m_\nu = \frac{f^2 m_D^2 m_N}{(m_E^2 + m_D^2)} F(x). \quad (13)$$

Since $F(0) = 1$ and goes to zero only as $x \rightarrow \infty$, this scenario does not favor a massless neutrino. If $f_{1,2,3}$ are all comparable in magnitude, the most likely outcome is three massive neutrinos with comparable masses.

Since the charged leptons also couple to $s_{1,2,3}$ through E^- , there is an unavoidable contribution to the muon anomalous magnetic moment given by [29]

$$\Delta a_\mu = \frac{(g-2)_\mu}{2} = \frac{f_\mu^2 m_\mu^2}{16\pi^2 m_E^2} \sum_i |U_{\mu i}|^2 G(x_i), \quad (14)$$

where

$$G(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1-x)^4}, \quad (15)$$

with $x_i = m_{si}^2/m_E^2$ and

$$U = O \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & i/\sqrt{2} & -i/\sqrt{2} \end{pmatrix}. \quad (16)$$

To get an estimate of this contribution, let $x_i \ll 1$, then $\Delta a_\mu = f_\mu^2 m_\mu^2 / 96\pi^2 m_E^2$. For $m_E \sim 1$ TeV, this is of order $10^{-11} f_\mu^2$, which is far below the present experimental sensitivity of 10^{-9} and can be safely ignored. The related amplitude for $\mu \rightarrow e\gamma$ is given by

$$A_{\mu e} = \frac{ef_\mu f_e m_\mu}{32\pi^2 m_E^2} \sum_i U_{ei}^* U_{\mu i} G(x_i). \quad (17)$$

Using the most recent $\mu \rightarrow e\gamma$ bound [30]

$$B = \frac{12\pi^2 |A_{\mu e}|^2}{m_\mu^2 G_F^2} < 5.7 \times 10^{-13}, \quad (18)$$

and the approximation $\sum_i U_{ei}^* U_{\mu i} G(x_i) \sim 1/36$ (based on tribimaximal mixing with $x_1 \sim 0$ and $x_2 \sim 1$) and $m_E \sim 1$ TeV, we find

$$f_\mu f_e < 0.03. \quad (19)$$

Let $f_{e,\mu,\tau} \sim 0.1$, $m_N \sim 10$ MeV, $m_D \sim 10$ GeV, $m_E \sim 1$ TeV, then the very reasonable scale of $m_\nu \sim 0.1$ eV in Eq. (7) is obtained, justifying its inverse seesaw origin.

Since N_L is the lightest particle with odd Z_2 , it is a would-be dark-matter candidate. However, suppose we add N_R so that the two pair up to have a large invariant Dirac mass,

then the lightest scalar (call it S) among $s_{1,2,3}$ is a dark-matter candidate. It interacts with the standard-model Higgs boson h according to

$$-\mathcal{L}_{int} = \frac{\lambda_{hS}}{2} v h S^2 + \frac{\lambda_{hS}}{4} h^2 S^2. \quad (20)$$

If we assume that all its other interactions are suppressed, then the annihilations $SS \rightarrow h \rightarrow$ SM particles and $SS \rightarrow hh$ determine its relic abundance, whereas its elastic scattering off nuclei via h exchange determines its possible direct detection in underground experiments. A detailed analysis [31] shows that the present limit of the invisible width of the observed 125 GeV particle (identified as h) allows m_S to be only within several GeV below $m_h/2$ or greater than about 150 GeV using the recent LUX data [32]. Note that the vector fermion doublet (E^0, E^-) is not the usually considered vector lepton doublet because it is odd under Z_2 and cannot mix with the known leptons.

In conclusion, we have shown how neutrino mass and dark matter may be connected using a one-loop mechanism proposed in 1998. This scotogenic model is naturally suited to implement the notion of inverse seesaw for neutrino mass, allowing the scale of new physics to be 1 TeV or less. The imposition of a softly broken Z_3 flavor symmetry yields an interesting pattern of radiative neutrino mass, allowing for maximal θ_{23} and maximal CP violation. The real singlet scalars in the dark sector carry lepton flavor, the lightest of which is absolutely stable. Our proposal provides thus a natural theoretical framework for this well-studied phenomenological possibility.

This work is supported in part by the U. S. Department of Energy under Grant No. de-sc0008541.

References

- [1] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).

- [2] E. Ma, Phys. Rev. Lett. **81**, 1171 (1998).
- [3] T. Yanagida, in *Proc. of the Workshop on Unified Theories and Baryon Number in the Universe* (KEK, Tsukuba, 1979), edited by O. Sawada and A. Sugamoto, p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p. 315; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); P. Minkowski, Phys. Lett. **67B**, 421 (1977).
- [4] J. Schechter and J. W. F. Valle, Phys. Rev. **D22**, 2227 (1980); M. Magg and C. Wetterich, Phys. Lett. **B94**, 61 (1980); T. P. Cheng and L.-F. Li, Phys. Rev. **D22**, 2860 (1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. **D23**, 165 (1981).
- [5] R. Foot, H. Lew, X.-G. He, and G. C. Joshi, Z. Phys. **C44**, 441 (1989).
- [6] A. Zee, Phys. Lett. **B93**, 389 (1980).
- [7] E. Ma, Phys. Rev. **D73**, 077301 (2006).
- [8] D. Restrepo, O. Zapata, and C. E. Yaguna, JHEP **1311**, 011 (2013).
- [9] D. Wyler and L. Wolfenstein, Nucl. Phys. **B218**, 205 (1983).
- [10] R. N. Mohapatra and J. W. F. Valle, Phys. Rev. **D34**, 1642 (1986).
- [11] E. Ma, Phys. Lett. **B191**, 287 (1987).
- [12] D. Aristizabal Sierra and C. E. Yaguna, JHEP **1108**, 013 (2011).
- [13] P. S. Bhupal Dev and A. Pilaftsis, Phys. Rev. **D86**, 113001 (2012).
- [14] E. Ma, Mod. Phys. Lett. **A24**, 2161 (2009).
- [15] A. de Gouvea *et al.*, arXiv:1310.4340 [hep-ex].

- [16] E. Ma, I. Picek, and B. Radovic, Phys. Lett. **B726**, 744 (2013).
- [17] T. Hambye, K. Kannike, E. Ma, and M. Raidal, Phys. Rev. **D75**, 095003 (2007).
- [18] H. Okada and T. Toma, Phys. Rev. **D86**, 033011 (2012).
- [19] E. Ma, Phys. Rev. **D80**, 013013 (2009).
- [20] F. Bazzocchi, Phys. Rev. **D83**, 093009 (2011).
- [21] S. S. C. Law and K. L. McDonald, Phys. Lett. **B713**, 490 (2012).
- [22] G. Guo, X.-G. He, and G.-N. Li, JHEP **1210**, 044 (2012).
- [23] ATLAS Collaboration, G. Aad *et al.*, Phys. Lett. **B716**, 1 (2012).
- [24] CMS Collaboration, S. Chatrchyan *et al.*, Phys. Lett. **B716**, 30 (2012).
- [25] E. Ma, Phys. Rev. Lett. **112**, 091801 (2014).
- [26] W. Grimus and L. Lavoura, Phys. Lett. **B579**, 113 (2004).
- [27] E. Ma, Phys. Rev. **D66**, 117301 (2002).
- [28] K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. **B552**, 207 (2003).
- [29] S. Kanemitsu and K. Tobe, Phys. Rev. **D86**, 095025 (2012).
- [30] MEG Collaboration, J. Adams *et al.*, Phys. Rev. Lett. **110**, 201801 (2013).
- [31] J. M. Cline, P. Scott, K. Kainulainen, and C. Weniger, Phys. Rev. **D88**, 055025 (2013).
- [32] LUX Collaboration, D. S. Akerib *et al.*, Phys. Rev. Lett. **112**, 091303 (2014).